

Practice Test J: Limits, Continuity, IVT

Name: _____

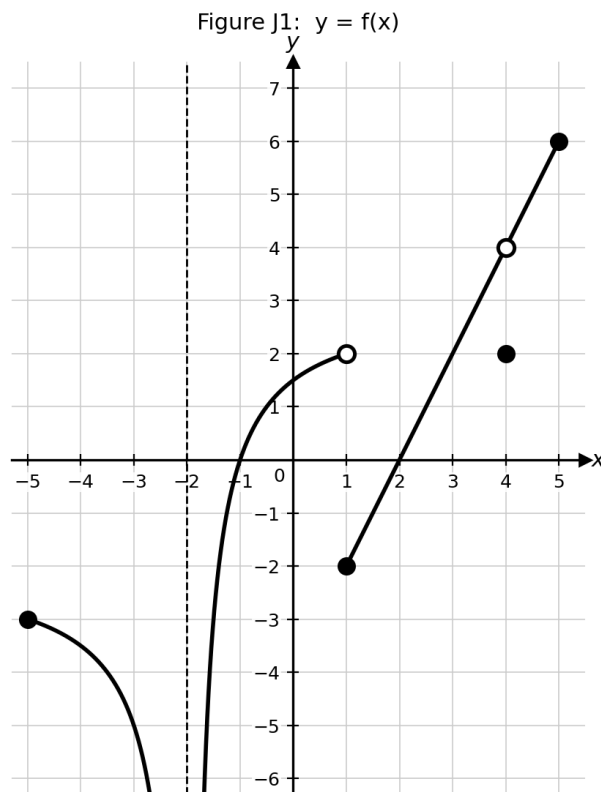
Time: 60 minutes No calculator 100 points

Answer exactly the question asked. Show work on every problem in Parts III–V.

If a limit is infinite, write ∞ or $-\infty$. If a limit does not exist for any other reason, write DNE.

If a function value does not exist, write *undefined*.

Part I. Graph. Questions 1(a)–1(g) refer to Figure J1.



1. Use the figure to find each value.

(a) $\lim_{x \rightarrow -2} f(x)$ _____

(b) $f(4)$ _____

(c) $\lim_{x \rightarrow 4} f(x)$ _____

(d) $\lim_{x \rightarrow -2^-} f(x)$ _____

(e) $\lim_{x \rightarrow 1^+} f(x)$ _____

(f) $\lim_{x \rightarrow 1} f(x)$ _____

(g) $f(-2)$ _____

(2 pts each, 14 pts)

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Part II. Table.

2. The table gives values of k . Assume the pattern continues as x approaches -2 . **(2 each, 6 pts)**

x	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
$k(x)$	4.51	4.0501	4.005001	8	3.995001	3.9501	3.51

- (a) $\lim_{x \rightarrow -2^+} k(x)$ _____ (b) $\lim_{x \rightarrow -2} k(x)$ _____ (c) $k(-2)$ _____

Part III. Algebra. Show work.

3. Evaluate $\lim_{t \rightarrow 3} \frac{t^2 + 3t - 4}{t^2 - 4t + 3}$, if it exists. **(5 pts)**

4. Evaluate $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 2}{x - 6}$, if it exists. **(5 pts)**

5. Evaluate $\lim_{x \rightarrow -3} \frac{x^3 + 2x^2 + 3x + 18}{x^2 + 4x + 3}$, if it exists. **(6 pts)**

6. Let $g(x) = \frac{x + 5}{x^2 + 2x - 3}$. Evaluate each limit. **(3 each, 9 pts)**

- (a) $\lim_{x \rightarrow -3^-} g(x)$ (b) $\lim_{x \rightarrow -3^+} g(x)$ (c) $\lim_{x \rightarrow -3} g(x)$

7. Let $p(x) = \begin{cases} x^2 + 2 & x \leq 2 \\ 3x + 2 & x > 2 \end{cases}$ (2 each, 10 pts)
- (a) $\lim_{x \rightarrow 6} p(x)$ (b) $\lim_{x \rightarrow 2^-} p(x)$ (c) $\lim_{x \rightarrow 2^+} p(x)$ (d) $\lim_{x \rightarrow 2} p(x)$ (e) $p(2)$

8. Let $v(x) = \frac{x^3 - 8}{x - 2}$. Find $v(2)$, if it is defined. (3 pts)

Part IV. Continuity. For 9 and 10, check each of the three conditions in the definition of continuity at a point and state your conclusion.

9. Using Figure J1, determine whether f is continuous at $x = 2$. (6 pts)

10. Let $h(x) = \begin{cases} 2x - 1 & x < 1 \\ -2x & x \geq 1 \end{cases}$
- Determine whether h is continuous at $x = 1$. (6 pts)

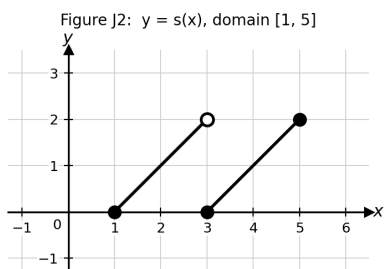
11. Using Figure J1, is f continuous on the closed interval $[-1, 1]$? Explain. (4 pts)

12. Let $u(x) = \frac{x-1}{x^2+3x}$. (3 each, 6 pts)
- (a) Is u continuous on $(0, 2]$? Explain. (b) Is u continuous on $[-2, 0]$? Explain.

Part V. Intermediate Value Theorem.

13. The function $g(x) = -x^3 + 2x^2 + x - 2$ is continuous on $[-1, 2]$. Find $g(-1)$. (2 pts)
14. Let $f(x) = x^2 + 5x$ on the interval $[1, 4]$. Does the Intermediate Value Theorem guarantee a number c in $(1, 4)$ with $f(c) = 24$? If it does, find every such c in $(1, 4)$. (6 pts)

15. Figure J2 shows the graph of s on its domain $[1, 5]$. Does the Intermediate Value Theorem guarantee a number c in $(1, 5)$ with $s(c) = 1$? If it does, find c . Explain. (4 pts)



16. True or False. Circle one. (4 each, 8 pts)
- (a) **T** **F** If f is continuous on $[1, 5]$, $f(1) = -5$, and $f(5) = 3$, then there is exactly one number c in $[1, 5]$ with $f(c) = 0$.
- (b) **T** **F** If g is continuous on $[1, 3]$, $g(1) = -3$, and $g(3) = 7$, then $g(c) = 0$ for at least one number c in $[1, 3]$.